

# Final states in small $x$ deep inelastic scattering <sup>1</sup>

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## Abstract

This talk summarises our work on the calculation of small- $x$  jet rates within the BFKL and CCFM approaches. The two approaches are proven to yield the same results at the leading logarithm level to order  $\bar{\alpha}_S^3$ . The proof is then extended to all orders.

## 1 Introduction.

An important feature of perturbative QCD is the coherent emission of soft gluons in deep inelastic scattering (DIS) at small Bjorken- $x$  [1]. This is specially relevant in the calculation of exclusive quantities, e.g. the number of gluons emitted in the final state. In DIS colour coherence leads to angular ordering with increasing opening angles towards the hard scale (the virtuality of the photon). If  $z_i$  is the fraction of energy of the  $(i-1)th$  gluon carried off by the  $i$ th gluon and  $q_i$  the transverse momentum then, in the limit  $z_i \ll 1$ , the angular ordering is implemented by the condition  $q_{i+1} > z_i q_i$ .

When the values of  $x$  are small enough logarithms in  $1/x$  need to be summed. This summation is performed by the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation which at leading order sums terms  $\sim [\alpha_S \ln(1/x)]^n$  [2]. Recently the next- to-leading terms have also been computed [3, 4].

The ( $t=0$ ) BFKL equation for  $f_\omega(\mathbf{k})$ , the unintegrated structure function in  $\omega$ -space ( $\omega$  is the variable conjugate to  $x$ ), expressed in a suitable form for exclusive quantities is [1, 5]:

$$f_\omega(\mathbf{k}) = f_\omega^0(\mathbf{k}) + \bar{\alpha}_S \int \frac{d^2 \mathbf{q}}{\pi q^2} \int_0^1 \frac{dz}{z} z^\omega \Delta_R(z, k) \Theta(q - \mu) f_\omega(\mathbf{q} + \mathbf{k}),$$

where  $\mu$  is a collinear cutoff,  $\mathbf{q}$  the transverse momentum of the emitted gluon, and  $\Delta_R(z_i, k_i) = \exp \left[ -\bar{\alpha}_S \ln \frac{1}{z_i} \ln \frac{k_i^2}{\mu^2} \right]$ , the gluon Regge factor which sums all the virtual contributions, with  $k_i \equiv |\mathbf{k}_i|$ , and  $\bar{\alpha}_S \equiv 3\alpha_S/\pi$ .

This form of the BFKL equation has a kernel which, under iteration, generates real gluon emissions with all the virtual corrections summed to all orders. As such, it is suitable for the study of the final state.

Defining the structure function  $F_{0\omega}(Q, \mu)$  by integrating over all  $\mu^2 \leq q_i^2 \leq Q^2$  we obtain:

$$F_{0\omega}(Q, \mu) = \Theta(Q - \mu) + \sum_{r=1}^{\infty} \int_{\mu^2}^{Q^2} \prod_{i=1}^r \frac{d^2 \mathbf{q}_i}{\pi q_i^2} dz_i \frac{\bar{\alpha}_S}{z_i} z_i^\omega \Delta_R(z_i, k_i) = 1 + \sum_{r=1}^{\infty} \sum_{n=r}^{\infty} C_0^{(r)}(n; T) \frac{\bar{\alpha}_S^n}{\omega^n},$$

with  $T = \ln(Q/\mu)$ .

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Modifying the BFKL formalism to account for coherence we get the ‘CCFM’ [1] expression for  $F_{0\omega}(Q, \mu)$ :

$$F_{\omega}(Q, \mu) = \Theta(Q - \mu) + \sum_{r=1}^{\infty} \int_0^{Q^2} \prod_{i=1}^r \frac{d^2 \mathbf{q}_i}{\pi q_i^2} dz_i \frac{\bar{\alpha}_S}{z_i} z_i^{\omega} \Delta(z_i, q_i, k_i) \Theta(q_i - z_{i-1} q_{i-1})$$

$$= 1 + \sum_{r=1}^{\infty} \sum_{n=r}^{\infty} \sum_{m=1}^n C^{(r)}(n, m; T) \frac{\bar{\alpha}_S^n}{\omega^{2n-m}},$$

where we introduced the coherence improved Regge factor  $\Delta(z_i, q_i, k_i) = \exp \left[ -\bar{\alpha}_S \ln \frac{1}{z_i} \ln \frac{k_i^2}{z_i q_i^2} \right]$ ;  $k_i > q_i$ , and for the first emission we take  $q_0 z_0 = \mu$ .

In the formalism with coherence no collinear cutoff is needed, except on the emission of the first gluon. This is because subsequent collinear emissions are regulated by the angular ordering constraint and it is those collinear emissions which induce the additional powers of  $1/\omega$ . In inclusive quantities the collinear singularities cancel. At a less inclusive level, such as for the associated distributions, the collinear singular terms need not cancel any more [5].

## 2 Equivalence of BFKL and CCFM approaches at order $\bar{\alpha}_S^3$ .

The rates for emission of fixed numbers of resolved final-state gluons, together with any number of unresolvable ones, were calculated in Ref. [6] in the leading logarithmic approximation, to third order in  $\bar{\alpha}_S$ . By resolved we mean having a transverse momentum larger than a resolution scale  $\mu_R$  constrained by the collinear cutoff and the hard scale,  $\mu \ll \mu_R \ll Q$ . Within the leading  $\log(1/x)$  approximation, the resolved gluons can be identified as jets since any corrections arising from additional radiation are suppressed by  $\mathcal{O}(\alpha_s)$ .

To calculate a given  $n$ -jet rate we consider all the graphs with  $n$  resolved gluons and any number of unresolvable ones. We expand the Regge factors to order  $\bar{\alpha}_S^3$ . Doing so we find that the jet rates expressions calculated both in the multi-Regge (BFKL) approach and in the coherent (CCFM) approach are the same. Namely:

$$\begin{aligned} \text{“0-jet”} &= \frac{(2\bar{\alpha}_S)}{\omega} S + \frac{(2\bar{\alpha}_S)^2}{\omega^2} \left[ \frac{S^2}{2} \right] + \frac{(2\bar{\alpha}_S)^3}{\omega^3} \left[ \frac{S^3}{6} \right] + \dots, \\ \text{“1-jet”} &= \frac{(2\bar{\alpha}_S)}{\omega} T + \frac{(2\bar{\alpha}_S)^2}{\omega^2} \left[ TS - \frac{1}{2} T^2 \right] + \frac{(2\bar{\alpha}_S)^3}{\omega^3} \left[ \frac{1}{3} T^3 - \frac{1}{2} T^2 S + \frac{1}{2} T S^2 \right] + \dots, \\ \text{“2-jet”} &= \frac{(2\bar{\alpha}_S)^2}{\omega^2} [T^2] + \frac{(2\bar{\alpha}_S)^3}{\omega^3} \left[ T^2 S - \frac{7}{6} T^3 \right] + \dots, \\ \text{“3-jet”} &= \frac{(2\bar{\alpha}_S)^3}{\omega^3} [T^3] + \dots, \end{aligned}$$

with  $T = \ln(Q/\mu_R)$  and  $S = \ln(\mu_R/\mu)$ . Note that in the calculation with coherence there exist stronger singularities ( $\omega \rightarrow 0$ ) than occur in the BFKL approach but these additional “coherence induced” logarithms cancel when we sum all the graphs contributing to the jet rates and the final results are identical to those obtained without coherence. We will see now how this cancellation persists for  $n$ -jet rates to all orders in  $\bar{\alpha}_S$ .

## 3 Equivalence of BFKL and CCFM approaches at all orders in $\bar{\alpha}_S$ .

In a recent paper [7] the work of Ref. [6] was extended to all orders, for any number of resolved gluons. The BFKL and CCFM formulations were shown to give the same jet rates in leading logarithmic approximation to all orders. The factorization of collinear singularities was demonstrated, and a simple generating function for the jet multiplicity distribution was obtained.

Working within the multi-Regge (BFKL) approach, a simple expression for the  $r$ -jet rate was found:

$$R_{\omega}^{(r \text{ jet})}(Q, \mu_R) = \frac{F_{\omega}^{(r \text{ jet})}(Q, \mu_R, \mu)}{F_{\omega}(Q, \mu)} = \frac{1}{r!} \frac{\partial^r}{\partial u^r} \left( \exp \left( -\frac{2\bar{\alpha}_s}{\omega} T \right) \left[ 1 + (1-u) \frac{2\bar{\alpha}_s}{\omega} T \right]^{\frac{u}{1-u}} \right) \Big|_{u=0}.$$

The same jet-rate generating function is obtained taking account of coherence from the CCFM formulation of small- $x$  dynamics. After convolution with the measured gluon structure function, it gives the predicted jet rates in the leading logarithmic region  $\ln(1/x) \gg T = \ln(Q/\mu_R) \gg 1$ , to all orders in  $\alpha_s$ , proving that the CCFM results for the multijet contributions are equal to the BFKL predictions, thus completing the all-orders extension of the results of Ref. [6].

From this generating function we can obtain quantities to all orders, for example the mean number of jets and the mean square fluctuation in this number,

$$\langle r \rangle = \frac{\partial}{\partial u} R_{\omega}(u, T) \Big|_{u=1} = \frac{2\bar{\alpha}_s}{\omega} T + \frac{1}{2} \left( \frac{2\bar{\alpha}_s}{\omega} T \right)^2, \quad \langle r^2 \rangle - \langle r \rangle^2 = \frac{2\bar{\alpha}_s}{\omega} T + \frac{3}{2} \left( \frac{2\bar{\alpha}_s}{\omega} T \right)^2 + \frac{2}{3} \left( \frac{2\bar{\alpha}_s}{\omega} T \right)^3.$$

In general, the  $p$ th central moment of the jet multiplicity distribution is a polynomial in  $\bar{\alpha}_s T/\omega$  of degree  $2p-1$ , indicating that the distribution becomes relatively narrow in the limit of very small  $x$  and large  $Q/\mu_R$ .

## 4 Conclusions.

It has been shown that at leading logarithmic level the BFKL and CCFM approaches give the same results for multi-jet rates in DIS at small  $x$ . Both methods give the same simple generating function for the jet rates. We will expect to find differences only at the sub-leading level and in more differential quantities such as multi-jet rapidity correlations [8].

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